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ABSTRACTS

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LIST OF TALKS

Alexander Allin
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An Analogue of Chvátal's Hamiltonicity Theorem for Randomly Perturbed Graphs

We consider Hamilton cycles in randomly perturbed graphs, that is, graphs obtained as the union of a deterministic graph H and a random graph $G(n, p)$. While most research into randomly perturbed graphs assumes a minimum degree condition on H, here we consider conditions on its degree sequence. Under the assumption of a degree sequence of H which is comparable with the classical condition of Chvátal (dependent on a parameter α analogous to the minimum degree condition in typical results in the area), we prove that there exists some constant $C = C(\alpha)$ such that taking $p = C/n$ suffices to a.a.s. obtain a Hamilton cycle in $H \cup G(n, p)$. Our result is best possible both in terms of the degree sequence condition and the asymptotic value of p, and extends the known results about Hamiltonicity in randomly perturbed graphs. We also provide results about pancyclicity under the same conditions.

This is joint work with Alberto Espuny Díaz.

Marcin Anholcer

Poznań University of Economics and Business

On Generalized Majority Colorings

A *majority coloring* of a directed graph is a vertex coloring in which each vertex has the same color as at most half of its out-neighbors. In this note we simplify some proof techniques and generalize previously known results on various variants of majority coloring. In particular, our unified and simple approach gives the best known results for:

- directed and undirected graphs,
- $\frac{1}{k}$-majority colorings (each vertex has the same color as at most $\frac{1}{k}$ of its out-neighbors),
- weighted edges,
- list colorings (choosability),
- on-line list colorings (paintability),
- non-uniform list lengths,
- *ranked* colors.

This is joint work with Bartłomiej Bosek, Jarosław Grytczuk, Grzegorz Gutowski, Jakub Przybyło and Mariusz Zając.

Maria Axenovich
Karlsruhe Institute of Technology

Unavoidable Order-Size Pairs in Graphs and Hypergraphs

A graph *has a pair* (m, f) if it has an induced subgraph on m vertices and f edges. We write $(n, e) \to (m, f)$ if any graph on n vertices and e edges has a pair (m, f). Let

$$S(n, m, f) = \{e : \ (n, e) \to (m, f)\} \text{ and}$$

$$\sigma(m, f) = \limsup_{n\to\infty} \frac{|S(n, m, f)|}{\binom{n}{2}}.$$

These notions were first introduced and investigated by Erdős, Füredi, Rothschild, and Sós. They found five pairs (m, f) with $\sigma(m, f) = 1$ and showed that for all other pairs $\sigma(m, f) \leq 2/3$. We extend these results in two directions.

First, in a joint work with Weber, we show that not only $\sigma(m, f)$ can be zero, but also $S(n, m, f)$ could be empty for some pairs (m, f) and any sufficiently large n. We call such pairs (m, f) *absolutely avoidable*.

Second, we consider a natural analogue $\sigma_r(m, f)$ of $\sigma(m, f)$ in the setting of r-uniform hypergraphs. Weber showed that for any $r \geq 3$ and $m > r$, $\sigma_r(m, f) = 0$ for most values of f. Surprisingly, it was not immediately clear whether there are nontrivial pairs (m, f), ($f \neq 0$, $f \neq \binom{m}{r}$, $r \geq 3$), for which $\sigma_r(m, f) > 0$. In a joint work with Balogh, Clemen, and Weber we show that $\sigma_3(6, 10) > 0$ and conjecture that in the 3-uniform case $(6, 10)$ is the only such pair.

Sebastian Babiński
Jagiellonian University

Graphs without Rainbow Paths with 3 Edges

One of the essential problems in extremal graph theory is the Turán problem asking to determine the maximum possible number of edges in graphs not containing a copy of a forbidden subgraph F. Research on this topic, and its various generalizations, led to development of many important theorems and lemmas applied far beyond graph theory.

Among possible generalizations of the Turán problem there is its rainbow version, which recently attracted high attention and started to be often considered. A natural way of defining the rainbow Turán problem is as follows. For fixed integer $k \geq 1$ and graph F we consider graphs $G_1, G_2, \ldots, G_k$ on a common set of vertices (each of them being interpreted as edges in a different color). The problem is to determine the maximum possible number of edges in each color such that there does not appear a rainbow copy of graph F, i.e., a copy of F which each edge belongs to a different graph G_i.

In my talk I consider this rainbow version of the Turán problem for F being a path. I present the tight asymptotic bound for the number of edges in the case of a path with 3 edges and any number of colors $k \geq 1$.

This is joint work with Andrzej Grzesik.

Maria Chudnovsky
Princeton University

Even Holes, Excluded Trees and Tree-Decompositions

Tree decompositions are a powerful tool in structural graph theory, that is traditionally used in the context of forbidden graph minors. In this talk we will describe several new results concerning induced subgraph obstructions to bounded tree-width. We will also outline the proof of the following result, obtained in joint work with Tara Abrishami, Bogdan Alecu, Sepehr Hajebi and Sophie Spirkl: for every tree T and integer t, there exists a constant c such that every even-hole-free graph with no induced subgraph isomorphic to T and no clique of size t has tree-width at most c.

Maciej Cisiński

AGH University of Science and Technology

Tree Packing Conjecture

The Gyárfás tree packing conjecture says that any set of trees on $2, 3, \ldots, n$ vertices has an edge-disjoint packing into complete graph on n vertices. Bollobás version of conjecture says that for every $k \geq 1$ there is $n_0(k)$ such that if $n > n_0(k)$, then every set of k trees $T_n, T_{n-1}, \ldots, T_{n-k+1}$ such that T_{n-i} has $n - j$ vertices pack into K_n. These versions of conjecture have partial results. Gyárfás's conjecture in case, where all trees are either path or star. Bollobás's conjecture with $k \leq 5$. In the talk we present some results and we focus on Bollobás's conjecture for almost-paths and almost-stars.

This is joint work with Andrzej Żak.

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Sebastian Czerwiński
University of Zielona Góra

A Harmonious Coloring and the Local Cut Lemma

A *harmonious coloring* of a hypergraph H is a vertex coloring such that it is a *rainbow coloring*, i.e. no two vertices on the same edge have the same color, and each subset of colors occurs on at most one edge. We present the application of the local cut lemma to the harmonious coloring of a hypergraph, see [2]. The local cut lemma is a generalization of the Lovász local lemma, the LCL was obtained by A. Bernshteyn, see [1].

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Dariusz Dereniowski

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Binary Search Generalizations in Graphs

We consider a problem of searching for an unknown target vertex t in a (possibly edge-weighted) graph. Each *vertex-query* points to a vertex v and the response either admits that v is the target or provides any neighbor s of v that lies on a shortest path from v to t. This model has been introduced for trees by Onak and Parys [1] and for general graphs by Emamjomeh-Zadeh et al. [2]. In this talk we discuss the error models (in which some responses may be incorrect) and our selected results in [3, 4, 5].

This is joint work with Aleksander Łukasiewicz, Stefan Tiegel, Przemysław Uznański, and Daniel Wolleb-Graf

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Kunal Dutta
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Uniform Brackets, Containers and Combinatorial Macbeath Regions

We study connections between three seemingly different combinatorial structures – *uniform brackets* in statistics and probability theory, *containers* in online and distributed learning theory, and *combinatorial Macbeath regions*, or *Mnets* in discrete and computational geometry. We show that these three concepts are manifestations of a single combinatorial property that can be expressed under a unified framework along the lines of Vapnik-Chervonenkis type theory for uniform convergence. These new connections help us to bring tools from discrete and computational geometry to prove improved bounds for these objects. Our improved bounds help to get an optimal algorithm for distributed learning of halfspaces, an improved algorithm for the distributed convex set disjointness problem, and improved regret bounds for online algorithms against σ-smoothed adversary for a large class of semi-algebraic threshold functions.

This is joint work with Arijit Ghosh and Shay Moran.

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Iryna Fryz
Vasyl' Stus Donetsk National University

An Algorithm for Constructing 24 Triple-Wise Orthogonal Latin Cubes

A Latin cube is a 3-dimensional table filled with elements of a set (a *carrier*) in such a way that each element of the carrier appears exactly once in each row of each dimension. A triplet of Latin cubes determined on the same carrier is called orthogonal, if under their superimposition we get a cube containing all triplets of the elements of the carrier. A set of n, $n > 3$, of Latin cubes are called *orthogonal* if each triplet of cubes from this set is orthogonal.

It is well-known that each Latin cube is a Cayley table of a ternary quasigroup. A ternary groupoid $(Q; f)$ is a *quasigroup* and f is called *invertible*, if each of the equations

$$f(x_1, a, b) = c, \qquad f(a, x_2, b) = c, \qquad f(a, b, x_3) = c$$

has a unique solution for all $a,\ b, c \in Q$. For every permutation $\sigma \in S_4$ a *σ-parastrophe* ${}^{\sigma}f$ of an invertible ternary operation f is defined by

$${}^{\sigma}f(x_{1\sigma}, x_{2\sigma}, x_{3\sigma}) = x_{4\sigma} :\Longleftrightarrow f(x_1, x_2, x_3) = x_4.$$

A quasigroup having maximal pairwise different parastrophes is called *asymmetric*. A ternary asymmetric quasigroup has $4! = 24$ parastrophes.

A triplet of ternary operations $f_1,\ f_2,\ f_3$ is called *orthogonal*, if for all $a_1, a_2, a_3 \in Q$ the system of equations

$$\begin{cases} f_1(x_1, x_2, x_3) = a_1, \\ f_2(x_1, x_2, x_3) = a_2, \\ f_3(x_1, x_2, x_3) = a_3 \end{cases}$$

has a unique solution.

There are only a few methods for constructing orthogonal Latin cubes [1], [2]. We propose an algorithm for constructing an asymmetric ternary quasigroup whose set of parastrophes are orthogonal. As a result, a set of 24 triple-wise orthogonal Latin cubes is obtained.

Earlier, the similar idea was realized by G. Belyavskaya and T. Popovich in [3]. They proposed an algorithm for constructing an asymmetric binary quasigroup and so 6 pairwise orthogonal Latin squares were obtained.

This is joint work with Fedir Sokhatsky.

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Przemysław Gordinowicz
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On n-Saturated Closed Graphs, or Randomness in the Service of Her Majesty Logic

One of the most important results in the theory of random graphs is given by Erdős and Rényi [1] probabilistic construction of countable universal homogeneous graph, called from this reason *the random graph*. The random graph is obtained, with probability 1, from the space $G(\mathbb{N}, p)$ where $p \in (0, 1)$ is fixed: vertices are natural numbers, any two are adjacent with probability p independently to the others. The key property is that it is unique $\aleph_0$-saturated countable graph: each possible one-vertex extension of any finite subgraph is realised in it.

Here we focus on topological graphs on the Cantor space 2^ω. Geschke [2] proved that there is a clopen graph on 2^ω which is 3-saturated (it realises each possible one-vertex extension of any subgraph of order at most 2), but the clopen graphs on 2^ω do not even have infinite subgraphs that are 4-saturated. It is also known that there is no closed graph on 2^ω which is $\aleph_0$-saturated. We complete this picture by proving that for every $n \in \mathbb{N}$ there is an n-saturated closed graph 2^ω [1]. The key lemma is based on a probabilistic argument. The final construction is an inverse limit of finite graphs.

This is joint work with Szymon Głąb.

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Aleksandra Gorzkowska
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Graphs with a Unique Maximum Independent Set up to Automorphisms

Given a graph G, we call a set $S \subseteq V(G)$ an *independent set* if no two vertices in S are adjacent. The maximum cardinality of an independent set in G is called the *independence number* of the graph G and is denoted $\alpha(G)$. An independent set with cardinality $\alpha(G)$ is called an *α-set*. We say that a graph G is α-unique if there is exactly one α-set in G. Hopkins and Staton and later Gunther, Hartnell and Rall characterized α-unique trees. Moreover, Gunther, Hartnell and Rall gave two equivalent conditions for a tree T to have exactly one independent set of cardinality $\alpha(T)$. Levit and Mandrescu extended that result to chordal graphs.

We say that a graph G is α-iso-unique if for any two α-sets S and S' in G there exists an automorphism φ of G such that $\varphi(S) = S'$. In this talk, we present results similar to the ones obtained by Gunther, Hartnell and Rall. In particular, we characterize all α-iso-unique trees. Moreover, we give partial results about chordal graphs and Cartesian products of graphs.

This is joint work with Boštjan Brešar, Tanja Dravec and Elżbieta Kleszcz.

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Martin Grohe

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The Algebra of Homomorphism Counts

Representations of graphs based on counting homomorphisms provide a surprisingly rich view on graphs with applications ranging from database theory to machine learning. Lovász (1967) showed that two graphs G and H are isomorphic if and only if they are homomorphism indistinguishable over the class of all graphs, i.e., for every graph F, the number of homomorphisms from F to G equals the number of homomorphisms from F to H. Recently, homomorphism indistinguishability over restricted classes of graphs such as bounded treewidth, bounded treedepth and planar graphs, has emerged as a surprisingly powerful framework for capturing diverse equivalence relations on graphs arising from logical equivalences and algebraic equation systems.

In this talk, I will introduce an algebraic framework for such results drawing from linear algebra and representation theory.

This is joint work with Holger Dell, Gaurav Rattan, and Tim Seppelt.

Jarosław Grytczuk
Warsaw University of Technology

Variations on Majority Coloring of Graphs

In every coloring of the vertices of a graph, some edges are *good* (properly colored), while others may be *bad* (monochromatic). In a *majority* coloring of a graph, every vertex shoud have at least as many good as bad edges among the ones it is incident to. It is well known and easy to prove that every *finite* simple graph has a majority coloring by using just *two* colors. It is not known however if the same is true for countable graphs (see [1], [7]).

Majority coloring can be considered for other combinatorial structures, like digraphs, hypergraphs, oriented hypergraphs, etc. For instance, in a majority coloring of a directed graph, every vertex should have at least as many good as bad among its *outgoing* edges. It is easy to prove that every finite digraph is majority 4-colorable, but it is conjectured that actually three colors should be sufficient (see [6]). A more general result for the *list* version of majority coloring of digraphs is proved in [3]. Other related results for majority *choosability* of graphs and digraphs are proved in [2], [4] and [5].

I will present some further problems and results concerning this topics.

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Igor Grzelec
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Conjectures about Local Irregularity of Graphs and 2-Multigraphs

We say that graph is *locally irregular* if adjacent vertices have different degrees. After short introduction about the well known 1-2-3 Conjecture [2] we discuss some results concerning the Local Irregularity Conjecture [1] and a new version of this conjecture proposed by Sedlar and Škrekovski in [3]. Next we present the Local Irregularity Conjecture for 2-*multigraphs*, which are multigraphs obtained from graphs by doubling each edge, and some results supporting this conjecture.

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Andrzej Grzesik

Jagiellonian University

Subgraph Densities in Graphs without a Forbidden Subgraph

A generalized Turán problem asks for two given graphs H and F what is the maximum number of copies of H in graphs not containing F as a subgraph. Despite many results for particular graphs, not much is known in general. Lidický and Murphy stated a conjecture providing conditions on H and F under which the maximum is asymptotically attained at a blow-up of a complete multipartite graph. In the talk we will present counterexamples to their conjecture and provide some alternative general conjectures. We also prove an asymptotically tight bound on the number of copies of any bipartite graph of radius at most 2 in triangle-free graphs.

This is joint work with Ervin Győri, Nika Salia and Casey Tompkins.

Grzegorz Gutowski
Jagiellonian University

Coloring Mixed Interval Graphs

A *mixed graph* has a set of vertices, a set of undirected edges, and a set of directed arcs. A *proper coloring* of a mixed graph G is a function c that assigns to each vertex in G a positive integer such that, for each edge $\{u, v\}$ in G, $c(u) \neq c(v)$ and, for each arc (u, v) in G, $c(u) < c(v)$. For a mixed graph G, the *chromatic number* $\chi(G)$ is the smallest number of colors in any proper coloring of G. A *directional interval graph* is a mixed graph whose vertices correspond to intervals on the real line. Such a graph has an edge between every two intervals where one is contained in the other and an arc between every two overlapping intervals, directed towards the interval that starts and ends to the right.

Coloring such graphs has applications in routing edges in layered orthogonal graph drawing according to the Sugiyama framework; the colors correspond to the tracks for routing the edges. We show how to recognize directional interval graphs, and how to compute their chromatic number efficiently. On the other hand, for *mixed interval graphs*, i.e., graphs where two intersecting intervals can be connected by an edge or by an arc in either direction arbitrarily, we prove that computing the chromatic number is NP-hard.

This is joint work with Florian Mittelstädt, Ignaz Rutter, Joachim Spoerhase, Alexander Wolff, and Johannes Zink.

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Hany Ibrahim
Mittweida University of Applied Sciences

Edge Contraction and Forbidden Induced Subgraph

A graph G is H-free if any subset of $V(G)$ does not induce a subgraph of G that is isomorphic to H. Given a graph H, we present sufficient and necessary conditions for a graph G such that G/e is H-free for any edge e in $E(G)$. Afterwards, we use these conditions to characterize forests, claw-free, $2K_2$-free, C_4-free, C_5-free, split graphs and matrogenic graphs.

Bartłomiej Kielak

Jagiellonian University

On Inducibility of Small Graphs and Oriented Graphs

Inducibility of a graph H is the limit of a maximum induced density of H over all graphs on n vertices, with n going to infinity. So far, it was determined only for very special classes of graphs and the smallest graph for which it is unknown is P_4, i.e. the path on 4 vertices. We improve the best known lower bound of inducibility of P_4 by providing an appropriate sequence of graphs with many copies of P_4.

We also consider the problem of determining the inducibility of oriented graphs on four vertices. We provide exact values for more than half of the graphs, and very close lower and upper bounds for all the remaining ones. It occurs that, for some graphs, the structure of extremal constructions maximizing density of its induced copies is very sophisticated and complex.

This is based on joint works with Andrzej Grzesik, Łukasz Bożyk, and Radosław Żak.

Hoang La
Jagellonian University

Feedback Vertex Sets in (Directed) Graphs of Bounded Degeneracy or Treewidth

We study the minimum size f of a feedback vertex set in directed and undirected n-vertex graphs of given degeneracy or treewidth. In the undirected setting the bound $\frac{k-1}{k+1}n$ is known to be tight for graphs with bounded treewidth k or bounded odd degeneracy k. We show that neither of the easy upper and lower bounds $\frac{k-1}{k+1}n$ and $\frac{k}{k+2}n$ can be exact for the case of even degeneracy. More precisely, for even degeneracy k we prove that $f < \frac{k}{k+2}n$ and that there exists a graph with $f \geq \frac{3k-2}{3k+4}n$.

For directed graphs of bounded degeneracy k, we prove that $f \leq \frac{k-1}{k+1}n$ and that this inequality is strict when k is odd. For directed graphs of bounded treewidth $k \geq 2$, we show that $f \leq \frac{k}{k+3}n$ and that there exists a graph with $f \geq \frac{k-2\lfloor\log_2(k)\rfloor}{k+1}n$. Further, we provide several constructions of low degeneracy or treewidth and large f.

This is a joint work with Kolja Knauer, and Petru Valicov.

Mikołaj Lewandowski
Poznań University of Technology

Two Disjoint Cycles in Digraphs

In 1963, Corrádi and Hajnal [3] proved that every undirected graph with at least $3k$ vertices and minimum degree at least $2k$ contains k vertex disjoint cycles. In 1981, Bermond and Thomassen [2] proposed an analogous conjecture for digraphs.

Conjecture. *For every positive integer k every digraph with minimum outdegree at least $2k-1$ contains k vertex disjoint cycles.*

For $k = 1$ the problem is easy and the case $k = 2$ was solved in 1983 by Thomassen [5]. More than two decades later Lichiardopol, Pór, and Sereni [4] managed to solve the case $k = 3$ and for all $k > 3$ the problem is wide open.

The existence of some finite integer $f(k)$ such that every digraph of minimum outdegree at least $f(k)$ contains k vertex disjoint cycles was established by Thomassen [5]. Later Alon [1] proved that it suffices to take $f(k) = 64k$.

We generalise the question asking for *all* outdegree sequences which force the existence of k vertex disjoint cycles and give the full answer for $k \leq 2$.

This is joint work with Joanna Polcyn and Christian Reiher.

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Adrian Michalski

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Some Properties of (1,2)-Dominating and Proper (1,2)-Dominating Sets in Graphs

Let $k \geq 1$ be an integer. A subset $D \subset V(G)$ is $(1,k)$-dominating if for every vertex $v \in V(G) \setminus D$ there are $u, w \in D$ such that $uv \in E(G)$ and $d_G(v,w) \leq k$. If $k = 1$ then we obtain the definition of $(1,1)$-dominating sets, which are also known as 2-dominating sets. If $k = 2$ then we have the concept of $(1,2)$-dominating sets, see [1]. A *proper* $(1,2)$*-dominating set* is a $(1,2)$-dominating set which is not $(1,1)$-dominating, see [3]. Although $(1,1)$-dominating sets and proper $(1,2)$-dominating sets cannot be equal, they do not have to be disjoint. Therefore, it is natural to ask what is the minimum possible number of vertices in the intersection of such sets in a given graph. This is why in [2] the $(1,\overline{2})$*-intersection index* of a graph was defined as the minimum cardinality of the intersection of a $(1,1)$-dominating set and a proper $(1,2)$-dominating set.

In the talk we present some relations between dominating sets, $(1,2)$-dominating sets and proper $(1,2)$-dominating sets, focusing mainly on a minimum cardinality of such sets in a given graph. Moreover, we give some results concerning the $(1,\overline{2})$-intersection index in some classes of graphs.

This is joint work with Anna Kosiorowska and Iwona Włoch.

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Yannick Mogge
Hamburg University of Technology

Walker-Breaker Games on Random Boards

The Maker-Breaker connectivity game and Hamilton cycle game belong to the best studied games in positional games theory, including results on biased games, games on random graphs, and fast winning strategies. Recently, the Connector-Breaker game variant, in which Connector has to claim edges such that her graph stays connected throughout the game, as well as the Walker-Breaker game variant, in which Walker has to claim her edges according to a walk, have received growing attention.

For instance, London and Pluhár [2] studied the threshold bias for the Connector-Breaker connectivity game on a complete graph K_n, and showed that there is a big difference between the cases when Maker's bias equals 1 or 2. Moreover, a recent result [1] shows that the threshold probability p for the $(2:2)$ Connector-Breaker connectivity game on a random graph $G \sim G_{n,p}$ is of order $n^{-2/3+o(1)}$. We extent this result further to Walker-Breaker games and prove that this probability is also enough for Walker to create a Hamilton cycle.

This is joint work with Dennis Clemens and Pranshu Gupta.

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Karolina Okrasa
Warsaw University of Technology

Computing Homomorphisms in Restricted Graph Classes

For a fixed graph H, the graph homomorphism problem $\textsc{Hom}(H)$ takes as an instance a graph G and asks whether there exists a homomorphism from G to H, i.e., a mapping $f : V(G) \to V(H)$ such that if uv is an edge of G, then $f(u)f(v)$ is an edge of H. The $\textsc{Hom}(H)$ problem is a generalization of the k-COLORING problem (they are equivalent if H is the complete graph on k vertices), which is arguably one of the best studied and well-known graph problems. Therefore, it is natural to try to extend results known for k-COLORING to the non-complete target graphs.

It turns out that the structural properties of input graphs may impact the complexity of determining the existence of certain homomorphisms. We are mainly interested in studying classes defined by forbidding specific structures. Another natural extension is analyzing how the complexity depends on some other structural parameter of the input instance, which may contain additional relevant information. For example, by Courcelle's theorem, we know that we can solve $\textsc{Hom}(H)$ in polynomial time in graph of bounded treewidth – but what is the exact time complexity of such algorithm?

In this talk we show how the elegant answers to this kind of questions can be provided using both, algebraic graph theory and combinatorial methods.

Natalia Paja

Rzeszów University of Technology

On $(2-d)$-Kernels in Two Generalizations of the Petersen Graph

A subset $D \subseteq V(G)$ is called a *p-dominating set* of a graph G if every vertex from $V(G) \setminus D$ has at least p neighbours in D. If $p = 1$, then we obtain *dominating set*. If $p = 2$, then we get *2-dominating set*.

A subset $J \subseteq V(G)$ is a $(2-d)$-*kernel* of a graph G if J is independent and 2-dominating, simultaneously.

In the talk we present two different generalizations of the Petersen graph and we give complete characterizations of these graphs which have $(2-d)$-kernel. Moreover, we determine the number of $(2-d)$-kernels of these graphs as well as their lower and upper kernel number.

This is joint work with Paweł Bednarz.

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Marta Piecyk

Warsaw University of Technology

Computing Graph Homomorphisms for Bounded-Cutwidth Graphs

For a fixed graph H, in the graph homomorphism problem, denoted by $\textsc{Hom}(H)$, we are given a graph G and we have to determine, whether there exists a *homomorphism* from G to H, i.e., an edge-preserving mapping $\varphi : V(G) \to V(H)$. In the list version of the problem, denoted by $\textsc{LHom}(H)$, the graph G is given along with lists $L : V(G) \to 2^{V(H)}$, and we ask if there exists a homomorphism φ from G to H that additionally respects lists, i.e., for every $v \in V(G)$ we have that $\varphi(v) \in L(v)$. Let us note that the graph homomorphism problem is a generalization of the well-known k-coloring problem.

For many graph parameters $f(G)$, the typical behavior is as follows. There is an algorithm solving k-coloring in time $k^{f(G)} \cdot |V(G)|^{O(1)}$ and this running time cannot be even slightly improved under standard complexity assumptions. This is not the case for a parameter called *cutwidth*, denoted by $ctw(G)$. Jansen and Nederlof [TCS 2019] provided a randomized algorithm that solves k-coloring in time $2^{ctw(G)} \cdot |V(G)|^{O(1)}$, and a deterministic one with running time $2^{\omega \cdot ctw(G)} \cdot |V(G)|^{O(1)}$, where ω is the matrix multiplication constant.

A natural question is if this behavior can be extended to a more general problem, i.e., (list) homomorphisms. It appears that there is no constant c such that for every H, the $\textsc{Hom}(H)$ problem can be solved in time $c^{ctw(G)} \cdot |V(G)|^{O(1)}$, unless the ETH fails. However, the tight complexity bounds of (list) homomorphism problem are not known.

This is joint work with Pavel Dvořak, Carla Groenland, Isja Mannes, Jesper Nederlof, and Paweł Rzążewski.

Oleg Pikhurko
University of Warwick

Hypergraph Turán Densities Can Have Arbitrarily Large Algebraic Degree

I will present our joint result with Xizhi Liu that, for every integer $r > 2$, the Turán densities of finite r-graph families can have arbitrarily large algebraic degrees. This answers a question of C. Grosu.

Tytus Pikies
Gdańsk University of Technology

Block Graphs and $\sum C_j$ Criterion

Consider a scheduling problem where the following input is given: a set of machines; a set of jobs; an optimality criterion; and a binary relation over the set of jobs. In this model no two jobs that are in the relation can be scheduled on the same machine.

When the relation forms a block graph the scheduling problem under the makespan criterion is closely related to the problem of equitable coloring of a block graph. Both the problems were given attention in a few recent papers, e.g. [1] and [2]. However, the problem of scheduling under $\sum C_j$ criterion was not investigated. Due to this, a proof that the problem is NP-hard even in a restricted case is presented. Moreover, an algorithm with a constant approximation ratio is presented as well.

This is a joint work with Krzysztof Giaro.

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Bartosz Podkanowicz

Jagiellonian University

Schnyder Labellings and Alon-Tarsi Numbers of Planar Graphs

We employ the technique of Schnyder labellings to derive a new simplified proof of the fact that the Alon-Tarsi number of a planar graph is at most 5. We discuss how this seemingly new approach is related to previous proofs that were based on the famous Thomassens argument for the choice number.

This is joint work with Jakub Kozik.

Adam Polak

EPFL

Fine-Grained Look at Graph and Matrix Problems

Algorithms research is usually concerned with how fast a given computational problem can be solved. However, without matching complexity lower bounds we cannot know if further speedups are possible or if the current algorithms are optimal. The classic conjecture that P is not NP is very successful at explaining why we cannot not find polynomial time algorithms for certain problems. However, it fails to address questions of the form "Given a problem with a cubic time algorithm, shall we expect to speed it up to quadratic or linear time?".

This motivates the area of fine-grained complexity, which studies sharper complexity hypotheses and reductions between problems with precise control over running times. A fine-grained reduction from a problem believed to be hard proves that the problem which we reduce to is also hard, and explains a reason for that hardness. More importantly, by studying reductions, we gain a better understanding of combinatorial structures involved in computational problems.

In this talk I will focus on fine-grained relations between a group of graph and matrix problems, dubbed "intermediate", whose time complexity situates them between "easy" matrix multiplication and "hard" All Pairs Shortest Paths. The technical part of the talk will be based on joint work with Andrea Lincoln and Virginia Vassilevska Williams (ITCS'20).

Magdalena Prorok

AGH University of Science and Technology

Distinguishing Symmetric Digraphs by Proper Arc-Colourings of Type I

A symmetric digraph $\overleftrightarrow{G}$ results from a graph G by converting each edge uv into a pair of arcs $\overrightarrow{uv}$ and $\overrightarrow{vu}$. We say that an arc-colouring c breaks an automorphism φ of $\overleftrightarrow{G}$ if there exists an arc $\overrightarrow{uv}$ whose colour is different from the colour of its image by φ. A colouring is distinguishing if it breaks all non-trivial automorphisms. A distinguishing chromatic index of a digraph $\overleftrightarrow{G}$ is the least number of colours in a distinguishing proper arc-colouring of $\overleftrightarrow{G}$.

There are several possible definitions of a proper colouring of a digraph since there are several possible definitions of adjacency of arcs. In this talk, we study the case when monochromatic 2-cycles and 2-paths are forbidden (in literature, this is usually called a proper colouring of type I). For such proper colourings, Poljak and Rödl [2] determined the chromatic index of a symmetric digraph $\overleftrightarrow{G}$ depending on the chromatic number of the underlying graph G. We prove optimal upper bounds for the distinguishing chromatic index of $\overleftrightarrow{G}$ with respect to the maximum degree of G.

This is joint work with Rafał Kalinowski, and Monika Pilśniak.

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Jakub Przybyło
AGH University of Science and Technology

Colouring Graphs from Triangle-Free List Assignments

We shall discuss an observation that Bernshteyn's proof [2] of the breakthrough result of Molloy [3] that triangle-free graphs are choosable from lists of size $(1+o(1))\Delta/\log\Delta$ can be adapted to yield a stronger result. In particular one may prove that such list sizes are sufficient to colour any graph of maximum degree Δ provided that vertices sharing a common colour in their lists do not induce a triangle in G, which encompasses all cases covered by Molloy's theorem. This was thus far known to be true for lists of size $(1000+o(1))\Delta/\log\Delta$, as implies a more general result due to Amini and Reed [1]. In the same vein, it can also proven that lists of length $2(r-2)\Delta\log_2\log_2\Delta/\log_2\Delta$ are sufficient if one replaces the triangle by any K_r with $r\geq 4$, which pushes slightly the multiplicative factor of $200r$ from Bernshteyn's result [2] down to $2(r-2)$. All bounds mentioned are also valid within the more general setting of correspondence colourings.

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Paweł Rzążewski
Warsaw University of Technology

Tree Decompositions with Bounded Independence Number: Beyond Independent Sets

Recently Dallard, Milanič, and Štorgel introduced and studied tree decompositions with bounded independence number: the size of a largest independent set contained in a single bag of the decomposition. Their motivation was to be able to solve Independent Set: given a graph with a tree decomposition of bounded independence number, this problem can be solved in polynomial time.

We extend the algorithmic applicability of tree decompositions with bounded independence number to other problems. First, we show that in polynomial time we can find a largest induced subgraph of bounded chromatic number, satisfying some prescribed CMSO_2-formula. Example of problem that can be expressed in this language are finding a largest induced tree (equivalent to Feedback Vertex Set) or finding a largest induced planar subgraph (equivalent to Planarization).

Second, we show that we can efficiently solve Distance-d Independent Set for even d. In stark contrast, for d odd the problem is NP-hard.

Quite surprisingly, similar phenomena can be observed in graphs with no long induced cycles.

This is joint work with Martin Milanič.

Michał T. Seweryn
Jagiellonian University

Three-dimensional Graph Products with Unbounded Stack-Number

The *stack-number* of a graph G is the least integer $s \geq 0$ for which there exist an ordering $(v_1, \ldots, v_N)$ of $V(G)$ and a function $\phi\colon E(G) \to \{1, \ldots, s\}$ such that for any pair of edges $v_i v_j, v_k v_\ell \in E(G)$ with $i < k < j < \ell$ we have $\phi(v_i v_j) \neq \phi(v_k v_\ell)$.

We show that the stack-number of the strong product $P_n \boxtimes P_n \boxtimes P_n$ of three n-vertex paths is $\Theta(n^{1/3})$. No non-trivial lower bound was previously known, and our result is the first explicit construction of graphs with bounded degree and unbounded stack-number. This is also the first example of graphs with bounded degree, bounded queue-number and unbounded stack-number.

Our proof uses topological and algebraic techniques: the main tool used in our proof of the lower bound is the topological overlap theorem of Gromov, while the upper bound uses a construction of permutations derived from Hadamard matrices. Finally, we use the geometry of the tesselation of $\mathbb{R}^3$ with truncated octahedra, to construct a family of graphs with all aforementioned properties of $P_n \boxtimes P_n \boxtimes P_n$ but with maximum degree 7 instead of 26.

This is joint work with David Eppstein, Robert Hickingbotham, Laura Merker, Sergey Norin, and David R. Wood.

Konstanty Junosza-Szaniawski
Warsaw University of Technology

Minimalization of Logic Formulas with Application in Cryptography

SAT-solvers continue to develop and they are able to solve bigger and bigger problems. Usually the shorter clauses are in CNF-form of SAT-formula the better solver can deal with it. Many problems can be encoded in different ways so natural question arises how to encode a problem into a SAT-formula in CNF-form such clauses are as short as possible.

We consider the problem of effective encoding of linear step of AES cypher. Given a homogeneous system of linear equations over field GF(2) we ask for an equivalent system with as short equations as possible (by a length of an equation we mean the number of non-zero coefficients). We can get an equivalent system by two operations: adding equations and by introducing new variable. We prove that the problem if a given linear system can be shorten to a system of equations with a given length is NP-hard. The problem has a nice combinatorial interpretation. Moreover we give some exact algorithm dedicated to this problem. Related problem was considered in [1].

This is joint work with Daniel Waszkiewicz.

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Ioannis Tasoulas
University of Piraeus

On the Hamiltonicity of Some Subgraphs in the Lattice of Binary Paths

Let $\mathcal{P}_n$, where n is a positive integer, be the set of all binary paths P of length $|P| = n$, i.e., lattice paths $P = p_1 p_2 \cdots p_n$ where each *step* p_i, $i \in [n]$, is either an *upstep* $u = (1, 1)$ or a *downstep* $d = (1, -1)$ and connects two consecutive points of the path P. The number of u's (resp. d's) in P is denoted by $|P|_u$ (resp. $|P|_d$). A maximal sequence of u's (resp. d's) in P is called *ascent* (resp. *descent*) of P. The last point of an ascent (resp. descent) is called *peak* (resp. *valley*) of the path. Clearly, every peak (resp. valley) corresponds to either an occurrence of ud (resp. du), or an occurrence of u (resp. d) at the end of the path. It is convenient to consider that the starting point of a path is the origin of a pair of axes. The y-coordinate of a lattice point on P is called *height* of this point.

A natural partial ordering on $\mathcal{P}_n$ is defined by the geometric representation of paths $P, Q \in \mathcal{P}_n$ where $P \leq Q$ whenever P lies (weakly) below Q. We note that Q covers P whenever Q is obtained from P by turning exactly one of P's valleys into a peak. It is well-known that the poset $(\mathcal{P}_n, \leq)$, or simply $\mathcal{P}_n$, is a finite, self-dual, distributive, graded lattice with minimum and maximum elements the paths $\mathbf{0}_n = d^n = \underbrace{dd \cdots d}_{n \text{ times}}$ and $\mathbf{1}_n = u^n = \underbrace{uu \cdots u}_{n \text{ times}}$ respectively. This lattice appears in the literature in various equivalent forms (e.g., sequences of integers [6], binary words [2, p. 92], subsets of $[n]$ [3], permutations of $[n]$ [8, p. 402], partitions of n into distinct parts [7], threshold graphs [4]).

In this work, we consider the Hasse graph G_n of $\mathcal{P}_n$, the edges of which are defined by the covering relation. The lattice $\mathcal{P}_n$ (resp. the graph G_n) is isomorphic to the lattice $M(n)$ (resp. the cover graph A_n of $M(n)$), introduced by Stanley [6] (resp. considered by Savage et al. [5]). Furthermore, in [5] (working on the isomorphic graph A_n) it is proved that for every $n \geq 3$ the subgraph $G_n \setminus \{d^n, d^{n-1}u, u^n, u^{n-1}d\}$ is Hamiltonian. Clearly, this is the largest Hamiltonian subgraph of G_n, since the excluded vertices do not belong to any cycle. In a similar direction, Eades and Hickey [1] gave a sufficient and necessary condition for the subgraph of G_n on the interval $[d^{n-k}u^k, u^k d^{n-k}]$ to have a Hamiltonian path (iff $k \leq 1$ or $k \geq n-1$ or n is even and k is odd).

In this work, it is shown that $G(P)$ is Hamiltonian for all paths P that have at least two peaks and ending with an upstep, where $G(P)$ is the subgraph of G_n induced by the interval $I(P) = [d^{n-2}ud, P]$, i.e., the interval which contains the elements of $\mathcal{P}_n$ less than or equal to P, excluding the first two elements of $\mathcal{P}_n$.

This is joint work with Kostas Manes, and Aristidis Sapounakis.

This work has been partly supported by the University of Piraeus Research Center

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István Tomon

ETH Zurich

Minimal Hyperplane Covers of Finite Spaces and Applications

At least how many hyperplanes are needed to cover the finite space $\mathbb{F}_p^n$? Clearly, the answer is p, as one can take p translations of any hyperplane. However, what if we also require that the normal vectors of the hyperplanes span the whole space, and none of the hyperplanes is redundant? This question turns out to be much more difficult, and is related to a number of long-standing conjectures, such as the Alon-Jaeger-Tarsi conjecture on non-vanishing linear maps, the Additive Basis conjecture, and a conjecture of Pyber about irredundant coset covers. I will present some progress on this question, which in particular resolves the first conjecture in a strong form, and the last conjecture as well. This is joint work with János Nagy and Péter Pál Pach.

Tom Trotter

Georgia Institute of Technology

Dimension and Posets with Planar Cover Graphs - Latest Results

In recent years, considerable attention has been given to two long standing conjectures: (1) Dimension is bounded in terms of the standard example number; and (2) Boolean dimension is bounded. We report on the most recent results, all obtained jointly with colleagues in the US and here in Poland. Since late February, I have been supported by a NAWA scholarship and working intensely with faculty and graduate students at Jagiellonian University.

Bartosz Walczak

Jagiellonian University

Coloring Ordered Graphs with Excluded Induced Ordered Subgraphs

A class of graphs is χ-*bounded* if the chromatic number of the graphs in the class is bounded by some function of the clique number. The well-known Gyárfás–Sumner conjecture asserts that the class of H-free graphs (i.e., graphs excluding H as an induced subgraph) is χ-bounded if and only if H is acyclic. An analogous question can be asked for *ordered graphs*, i.e., graphs equipped with a total order on the vertices. Say that an ordered graph H is χ-*bounding* if the class of H-free ordered graphs (i.e., ordered graphs excluding H as an induced ordered subgraph) is χ-bounded. So which ordered graphs are χ-bounding?

In joint work with Piotr Mikołajczyk, we prove that a connected ordered graph is χ-bounding if and only if it is a star, and we characterize the crossing-free ordered graphs that are χ-bounding. In joint work with Marcin Briański and James Davies, we prove that every ordered matching is χ-bounding, which confirms a conjecture by István Tomon.

Coloring ordered graphs with excluded (induced) ordered subgraphs has been little explored so far. In this talk, I will introduce the audience to this topic (and, in particular, to the above-mentioned results) with a focus on missing cases that prevent us from stating or conjecturing a full characterization of χ-bounding ordered graphs.

Karol Węgrzycki
Saarland University

Recent Progress on Subset Sum via Additive Combinatorics

In the Subset Sum problem, we are given a set of n integers and the task is to decide if any subset of these integers sums up to 0. I will talk about a recent progress on algorithms for the Subset Sum problem. I will focus on different regimes of parameters for this problem. In the "dense regime," I will present methods that allow us to exploit the tools based on additive-combinatorics to improve the currently best approximation and pseudopolynomial time algorithms. I will also sketch the main additive structure used to improve upon the currently best algorithms for "sparse regime"

Michał Zwierzyński
Warsaw University of Technology

Parallel Hiking Trails in the Mountains on the Torus

Let's consider a continuous function $f : [0,1] \to \mathbb{R}$ such that $f(0) = f(1)$. Let's assume that f has finitely many local extrema and the values at these extrema are pairwise different non-integers modulo 1. Then, let $\hat{f}(x) = f(x) \bmod 1$ for $x \in [0,1]$. The graph of $\hat{f}$ we will call a *mountain*. Note that $\hat{f} : [0,1] \to [0,1]$ and if we glue the endpoints of the interval $[0,1]$, the graph of $\hat{f}$ is the subset of the torus. See Figure 1 for an example of a mountain.

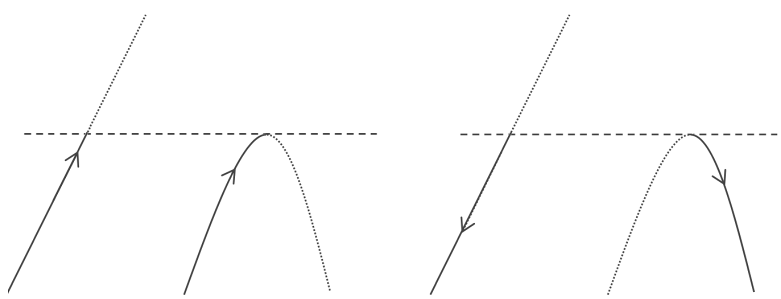

Figure 1: An example of a mountain, the hikers A and B, a peak s_0 and a valley s_1.

Now, let's imagine two points (the *hikers*) on a mountain m which is the graph of $\hat{f}$. The hikers can move on m but all the time they must be on the same level on m (like in Figure 1).

Now, we will add more restrictions for moving in the mountains. The hikers always move forward (left or right) except when one of them encounters a peak or valley (an extremum of $\hat{f}$) – then the other hiker turns back (see Figure 2). Thanks to this strange procedure, the hikers will be on the same level all the time.

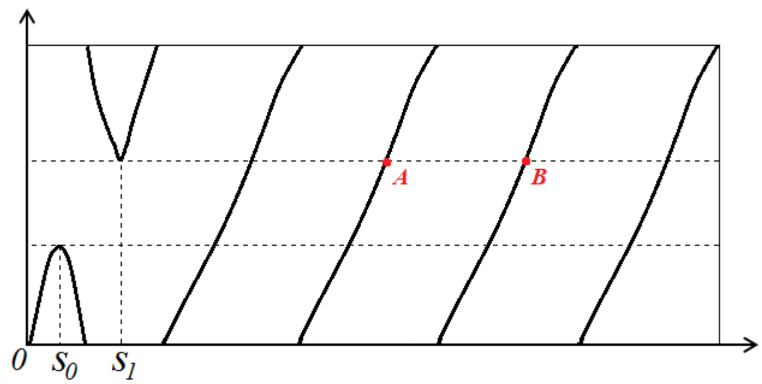

Figure 2: Turning back.

Finally, we are asking: how many different hiking trails are for a fixed mountain? If both hikers starts in the same peak of valley – will they ever meet again (if so, where?)? During the talk, we will get to know partial answers to the questions posed.

If we look at the list of articles below, we can get scared by the subject of differential geometry. This is because the problem described is related to the geometric properties of many sets such as the Wigner caustic, the Centre Symmetry Set, the secant caustic, the constant measure set. Any parallel hiking trail property found will result in some global geometry of said sets. However, during the presentation, we will only focus on mountain paths in isolation from differential geometry.

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