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Unavoidable Order-Size Pairs in Graphs and Hypergraphs

A graph *has a pair* (m, f) if it has an induced subgraph on m vertices and f edges. We write $(n, e) \to (m, f)$ if any graph on n vertices and e edges has a pair (m, f). Let

$$S(n, m, f) = \{e : \ (n, e) \to (m, f)\} \text{ and}$$

$$\sigma(m, f) = \limsup_{n \to \infty} \frac{|S(n, m, f)|}{\binom{n}{2}}.$$

These notions were first introduced and investigated by Erdős, Füredi, Rothschild, and Sós. They found five pairs (m, f) with $\sigma(m, f) = 1$ and showed that for all other pairs $\sigma(m, f) \leq 2/3$. We extend these results in two directions.

First, in a joint work with Weber, we show that not only $\sigma(m, f)$ can be zero, but also $S(n, m, f)$ could be empty for some pairs (m, f) and any sufficiently large n. We call such pairs (m, f) *absolutely avoidable.*

Second, we consider a natural analogue $\sigma_r(m, f)$ of $\sigma(m, f)$ in the setting of r-uniform hypergraphs. Weber showed that for any $r \geq 3$ and $m > r$, $\sigma_r(m, f) = 0$ for most values of f. Surprisingly, it was not immediately clear whether there are nontrivial pairs (m, f), $(f \neq 0,\ f \neq \binom{m}{r},\ r \geq 3)$, for which $\sigma_r(m, f) > 0$. In a joint work with Balogh, Clemen, and Weber we show that $\sigma_3(6, 10) > 0$ and conjecture that in the 3-uniform case $(6, 10)$ is the only such pair.