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An Algorithm for Constructing 24 Triple-Wise Orthogonal Latin Cubes

A Latin cube is a 3-dimensional table filled with elements of a set (a *carrier*) in such a way that each element of the carrier appears exactly once in each row of each dimension. A triplet of Latin cubes determined on the same carrier is called orthogonal, if under their superimposition we get a cube containing all triplets of the elements of the carrier. A set of n, $n > 3$, of Latin cubes are called *orthogonal* if each triplet of cubes from this set is orthogonal.

It is well-known that each Latin cube is a Cayley table of a ternary quasigroup. A ternary groupoid $(Q; f)$ is a *quasigroup* and f is called *invertible*, if each of the equations

$$f(x_1, a, b) = c, \qquad f(a, x_2, b) = c, \qquad f(a, b, x_3) = c$$

has a unique solution for all a, $b, c \in Q$. For every permutation $\sigma \in S_4$ a *σ-parastrophe* ${}^{\sigma}f$ of an invertible ternary operation f is defined by

$${}^{\sigma}f(x_{1\sigma}, x_{2\sigma}, x_{3\sigma}) = x_{4\sigma} :\Longleftrightarrow f(x_1, x_2, x_3) = x_4.$$

A quasigroup having maximal pairwise different parastrophes is called *asymmetric*. A ternary asymmetric quasigroup has $4! = 24$ parastrophes.

A triplet of ternary operations f_1, f_2, f_3 is called *orthogonal*, if for all $a_1, a_2, a_3 \in Q$ the system of equations

$$\begin{cases} f_1(x_1, x_2, x_3) = a_1, \\ f_2(x_1, x_2, x_3) = a_2, \\ f_3(x_1, x_2, x_3) = a_3 \end{cases}$$

has a unique solution.

There are only a few methods for constructing orthogonal Latin cubes [1], [2]. We propose an algorithm for constructing an asymmetric ternary quasigroup whose set of parastrophes are orthogonal. As a result, a set of 24 triple-wise orthogonal Latin cubes is obtained.

Earlier, the similar idea was realized by G. Belyavskaya and T. Popovich in [3]. They proposed an algorithm for constructing an asymmetric binary quasigroup and so 6 pairwise orthogonal Latin squares were obtained.

This is joint work with Fedir Sokhatsky.

References

[1] T. Evans, *The construction of orthogonal k-skeins and Latin k-cubes*, Aequationes Mathematicae 13(3), 1976, pp. 485–491.

[2] M. Trenkler, *On orthogonal Latin p-dimensional cubes*, Czechoslovak Mathematical Journal 55(3), 2005, pp. 725–728.

[3] G.B. Belyavskaya, T.V. Popovich, *Totally conjugate orthogonal quasigroups and complete graphs*, Fundamentalnaya i prikladnaya matematika 13(8), 2010, pp. 17–26. (in Russian)