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On n-Saturated Closed Graphs, or Randomness in the Service of Her Majesty Logic

One of the most important results in the theory of random graphs is given by Erdős and Rényi [1] probabilistic construction of countable universal homogeneous graph, called from this reason *the random graph*. The random graph is obtained, with probability 1, from the space $G(\mathbb{N}, p)$ where $p \in (0,1)$ is fixed: vertices are natural numbers, any two are adjacent with probability p independently to the others. The key property is that it is unique $\aleph_0$-saturated countable graph: each possible one-vertex extension of any finite subgraph is realised in it.

Here we focus on topological graphs on the Cantor space 2^ω. Geschke [2] proved that there is a clopen graph on 2^ω which is 3-saturated (it realises each possible one-vertex extension of any subgraph of order at most 2), but the clopen graphs on 2^ω do not even have infinite subgraphs that are 4-saturated. It is also known that there is no closed graph on 2^ω which is $\aleph_0$-saturated. We complete this picture by proving that for every $n \in \mathbb{N}$ there is an n-saturated closed graph 2^ω [3]. The key lemma is based on a probabilistic argument. The final construction is an inverse limit of finite graphs.

This is joint work with Szymon Głąb.

References

[1] P. Erdős, A. Rényi, *Asymmetric graphs*, Acta Mathematica Academiae Scientiarum Hungaricae 14, 1963, pp. 295–315.

[2] S. Geschke, *Clopen Graphs*, Fundamenta Mathematicae 220, 2013, pp. 155–189.

[3] S. Głąb, P. Gordinowicz, *On n-saturated closed graphs*, Results in Mathematics 77, 2022, #184, 6pp .