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Graphs with a Unique Maximum Independent Set up to Automorphisms

Given a graph G, we call a set $S \subseteq V(G)$ an *independent set* if no two vertices in S are adjacent. The maximum cardinality of an independent set in G is called the *independence number* of the graph G and is denoted $\alpha(G)$. An independent set with cardinality $\alpha(G)$ is called an *α-set*. We say that a graph G is α-unique if there is exactly one α-set in G. Hopkins and Staton and later Gunther, Hartnell and Rall characterized α-unique trees. Moreover, Gunther, Hartnell and Rall gave two equivalent conditions for a tree T to have exactly one independent set of cardinality $\alpha(T)$. Levit and Mandrescu extended that result to chordal graphs.

We say that a graph G is α-iso-unique if for any two α-sets S and S' in G there exists an automorphism φ of G such that $\varphi(S) = S'$. In this talk, we present results similar to the ones obtained by Gunther, Hartnell and Rall. In particular, we characterize all α-iso-unique trees. Moreover, we give partial results about chordal graphs and Cartesian products of graphs.

This is joint work with Boštjan Brešar, Tanja Dravec and Elżbieta Kleszcz.

References

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