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Coloring Mixed Interval Graphs

A *mixed graph* has a set of vertices, a set of undirected edges, and a set of directed arcs. A *proper coloring* of a mixed graph G is a function c that assigns to each vertex in G a positive integer such that, for each edge $\{u, v\}$ in G, $c(u) \neq c(v)$ and, for each arc (u, v) in G, $c(u) < c(v)$. For a mixed graph G, the *chromatic number* $\chi(G)$ is the smallest number of colors in any proper coloring of G. A *directional interval graph* is a mixed graph whose vertices correspond to intervals on the real line. Such a graph has an edge between every two intervals where one is contained in the other and an arc between every two overlapping intervals, directed towards the interval that starts and ends to the right.

Coloring such graphs has applications in routing edges in layered orthogonal graph drawing according to the Sugiyama framework; the colors correspond to the tracks for routing the edges. We show how to recognize directional interval graphs, and how to compute their chromatic number efficiently. On the other hand, for *mixed interval graphs*, i.e., graphs where two intersecting intervals can be connected by an edge or by an arc in either direction arbitrarily, we prove that computing the chromatic number is NP-hard.

This is joint work with Florian Mittelstädt, Ignaz Rutter, Joachim Spoerhase, Alexander Wolff, and Johannes Zink.

References

[1] G. Gutowski, F. Mittelstädt, I. Rutter, J. Spoerhase, A. Wolff, J. Zink. Coloring Mixed and Directional Interval Graphs. *Graph Drawing 2022*. URL: `https://arxiv.org/abs/2208.14250`.