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Two Disjoint Cycles in Digraphs

In 1963, Corrádi and Hajnal [3] proved that every undirected graph with at least $3k$ vertices and minimum degree at least $2k$ contains k vertex disjoint cycles. In 1981, Bermond and Thomassen [2] proposed an analogous conjecture for digraphs.

Conjecture. *For every positive integer k every digraph with minimum outdegree at least $2k-1$ contains k vertex disjoint cycles.*

For $k=1$ the problem is easy and the case $k=2$ was solved in 1983 by Thomassen [5]. More than two decades later Lichiardopol, Pór, and Sereni [4] managed to solve the case $k=3$ and for all $k>3$ the problem is wide open.

The existence of some finite integer $f(k)$ such that every digraph of minimum outdegree at least $f(k)$ contains k vertex disjoint cycles was established by Thomassen [5]. Later Alon [1] proved that it suffices to take $f(k)=64k$.

We generalise the question asking for *all* outdegree sequences which force the existence of k vertex disjoint cycles and give the full answer for $k \leq 2$.

This is joint work with Joanna Polcyn and Christian Reiher.

References

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