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COMPUTING HOMOMORPHISMS IN RESTRICTED GRAPH CLASSES

For a fixed graph H , the graph homomorphism problem $\text{HOM}(H)$ takes as an instance a graph G and asks whether there exists a homomorphism from G to H , i.e., a mapping $f : V(G) \rightarrow V(H)$ such that if uv is an edge of G , then $f(u)f(v)$ is an edge of H . The $\text{HOM}(H)$ problem is a generalization of the k -COLORING problem (they are equivalent if H is the complete graph on k vertices), which is arguably one of the best studied and well-known graph problems. Therefore, it is natural to try to extend results known for k -COLORING to the non-complete target graphs.

It turns out that the structural properties of input graphs may impact the complexity of determining the existence of certain homomorphisms. We are mainly interested in studying classes defined by forbidding specific structures. Another natural extension is analyzing how the complexity depends on some other structural parameter of the input instance, which may contain additional relevant information. For example, by Courcelle's theorem, we know that we can solve $\text{HOM}(H)$ in polynomial time in graph of bounded treewidth – but what is the exact time complexity of such algorithm?

In this talk we show how the elegant answers to this kind of questions can be provided using both, algebraic graph theory and combinatorial methods.