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Computing Graph Homomorphisms for Bounded-Cutwidth Graphs

For a fixed graph H, in the graph homomorphism problem, denoted by $\textsc{Hom}(H)$, we are given a graph G and we have to determine, whether there exists a *homomorphism* from G to H, i.e., an edge-preserving mapping $\varphi : V(G) \to V(H)$. In the list version of the problem, denoted by $\textsc{LHom}(H)$, the graph G is given along with lists $L : V(G) \to 2^{V(H)}$, and we ask if there exists a homomorphism φ from G to H that additionally respects lists, i.e., for every $v \in V(G)$ we have that $\varphi(v) \in L(v)$. Let us note that the graph homomorphism problem is a generalization of the well-known k-coloring problem.

For many graph parameters $f(G)$, the typical behavior is as follows. There is an algorithm solving k-coloring in time $k^{f(G)} \cdot |V(G)|^{O(1)}$ and this running time cannot be even slightly improved under standard complexity assumptions. This is not the case for a parameter called *cutwidth*, denoted by $ctw(G)$. Jansen and Nederlof [TCS 2019] provided a randomized algorithm that solves k-coloring in time $2^{ctw(G)} \cdot |V(G)|^{O(1)}$, and a deterministic one with running time $2^{\omega \cdot ctw(G)} \cdot |V(G)|^{O(1)}$, where ω is the matrix multiplication constant.

A natural question is if this behavior can be extended to a more general problem, i.e., (list) homomorphisms. It appears that there is no constant c such that for every H, the $\textsc{Hom}(H)$ problem can be solved in time $c^{ctw(G)} \cdot |V(G)|^{O(1)}$, unless the ETH fails. However, the tight complexity bounds of (list) homomorphism problem are not known.

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