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Three-dimensional Graph Products with Unbounded Stack-Number

The *stack-number* of a graph G is the least integer $s \geq 0$ for which there exist an ordering $(v_1, \ldots, v_N)$ of $V(G)$ and a function $\phi\colon E(G) \to \{1, \ldots, s\}$ such that for any pair of edges $v_i v_j, v_k v_\ell \in E(G)$ with $i < k < j < \ell$ we have $\phi(v_i v_j) \neq \phi(v_k v_\ell)$.

We show that the stack-number of the strong product $P_n \boxtimes P_n \boxtimes P_n$ of three n-vertex paths is $\Theta(n^{1/3})$. No non-trivial lower bound was previously known, and our result is the first explicit construction of graphs with bounded degree and unbounded stack-number. This is also the first example of graphs with bounded degree, bounded queue-number and unbounded stack-number.

Our proof uses topological and algebraic techniques: the main tool used in our proof of the lower bound is the topological overlap theorem of Gromov, while the upper bound uses a construction of permutations derived from Hadamard matrices. Finally, we use the geometry of the tesselation of $\mathbb{R}^3$ with truncated octahedra, to construct a family of graphs with all aforementioned properties of $P_n \boxtimes P_n \boxtimes P_n$ but with maximum degree 7 instead of 26.

This is joint work with David Eppstein, Robert Hickingbotham, Laura Merker, Sergey Norin, and David R. Wood.