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Coloring Ordered Graphs with Excluded Induced Ordered Subgraphs

A class of graphs is χ-*bounded* if the chromatic number of the graphs in the class is bounded by some function of the clique number. The well-known Gyárfás–Sumner conjecture asserts that the class of H-free graphs (i.e., graphs excluding H as an induced subgraph) is χ-bounded if and only if H is acyclic. An analogous question can be asked for *ordered graphs*, i.e., graphs equipped with a total order on the vertices. Say that an ordered graph H is χ-*bounding* if the class of H-free ordered graphs (i.e., ordered graphs excluding H as an induced ordered subgraph) is χ-bounded. So which ordered graphs are χ-bounding?

In joint work with Piotr Mikołajczyk, we prove that a connected ordered graph is χ-bounding if and only if it is a star, and we characterize the crossing-free ordered graphs that are χ-bounding. In joint work with Marcin Briański and James Davies, we prove that every ordered matching is χ-bounding, which confirms a conjecture by István Tomon.

Coloring ordered graphs with excluded (induced) ordered subgraphs has been little explored so far. In this talk, I will introduce the audience to this topic (and, in particular, to the above-mentioned results) with a focus on missing cases that prevent us from stating or conjecturing a full characterization of χ-bounding ordered graphs.